



Multiphysics modeling of a micro-scale Stirling refrigeration system



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ABSTRACT

A new micro-scale refrigeration system composed of arrays of silicon MEMS cooling elements that operate on the Stirling cycle has been designed. In this paper, we describe a multiphysics computational approach for analyzing the system performance that considers compressible flow and heat transfer with a large deformable mesh. The regenerator pressure drop and effectiveness are first explored to determine the optimal porosity. A value near 0.9 is found to maximize the coefficient of performance. To overcome the computational complexity brought about by the fine pillar structure in the regenerator, a porous medium model is used to allow for modeling of a full element. Parametric studies demonstrate the effect of the operating frequency on the cooling capacity and the coefficient of performance.

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1. Introduction

Micro-scale coolers have a wide range of potential applications, such as cooling chip- and board-level electronics, sensors, and radio-frequency systems [1]. The coefficient of performance of the cooler (COP), is defined as the ratio of heat removal, Q , to the input work, W , i.e., $COP = (Q/W)$. The Carnot coefficient of performance, COP_C , which represents the maximum theoretical efficiency possible between constant temperature hot (T_H) and cold (T_C) sources, can be found from $COP_C = T_C/(T_H - T_C)$.

Thermoelectric coolers are a common refrigeration device and have been scaled to the micro-domain [2–5]. The efficiency of thermoelectric energy conversion is determined by the thermoelectric material's figure of merit, ZT , which is defined by $(S^2\sigma T/k)$ where S , σ and k are the Seebeck coefficient, electrical conductivity, and thermal conductivity [6]. Significant challenges exist to increase ZT beyond unity due to the difficulty of reducing the thermal conductivity while maintaining good electrical properties [7]. The maximum COP of a thermoelectric cooler is [2]

$$COP = \frac{T_C (\sqrt{1 + ZT} - T_H/T_C)}{(T_H - T_C) (\sqrt{1 + ZT} + 1)} \quad (1)$$

This maximum COP is plotted as a function of the temperature difference ($T_H - T_C$) in Fig. 1 for ZT values of one and two (the latter is the maximum value currently available in bulk thermoelectric materials [2]). For a typical temperature difference of 25 K for cooling in ambient conditions, the maximum COP of a realistic thermoelectric cooler is 2.6, which is 24% of the Carnot COP .

In recent years, novel solid-state cooling technologies, such as electrocaloric and magnetocaloric cooling, have attracted interest [8,9]. The electrocaloric effect is a phenomenon in which reversible, polarization-related temperature and entropy changes appear under the application and removal of an electric field. Potential application of this technology is limited, however, by the relatively low temperature and entropy changes at ambient conditions for most ferroelectric materials [8]. The magnetocaloric effect is similar to the electrocaloric effect, with the use of a magnetic field instead of an electric field. The high magnetic fields required for magnetocaloric cooling are difficult to realize, especially at the micro-scale [9].

Micro-scale devices operating on the Stirling cycle, which was developed in 1816 [10], are an attractive potential alternative due to the high efficiencies realized for macro-scale Stirling machines [11].

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Nomenclature

COP	coefficient of performance
COP_C	Carnot COP
D_h	hydraulic diameter, m
H	height, m
L	length, m
M	amplitude of the mass flow rate, kg/s
M_g	molar mass of the gas, kg/mol
N	number of pillars in the computational domain
Nu_{D_h}	Nusselt number based on the hydraulic diameter
Q	heat removal, J
R	ideal gas constant, J/(mol K)
Re_{D_h}	Reynolds number based on the hydraulic diameter
S	Seebeck coefficient, V/K
S_L	horizontal pitch, m
S_T	vertical pitch, m
T	absolute temperature, K
V	volume, mm ³
W	work, J
Z_{max}	vibration amplitude of the diaphragm, m
ZT	figure of merit
a_{fs}	solid surface area per unit volume, 1/m
c_p	specific heat, J/(kg K)
f	frequency, Hz
h_{fs}	convection coefficient, W/(m ² K)
k	thermal conductivity, W/(m K)
m	mass, kg
$\dot{m}$	mass flow rate, kg/s

p	pressure, Pa
q	cooling capacity, W/cm ²
t	time, s
$\mathbf{u}$	velocity vector, m/s
u	velocity, m/s
$\bar{u}$	average velocity, m/s
Δp	pressure drop, Pa
β_F	Forchheimer coefficient, kg/m ⁴
ε	porosity
η	regenerator effectiveness
κ	permeability, m ²
μ	dynamic viscosity, Pa s
ρ	density, kg/m ³
σ	electrical conductivity, S/m
$\bar{\tau}$	viscous tensor, N/m ²

Subscripts

C	cold side
H	hot side
g	gas
loss	loss
s	solid
out	outlet
r	regenerator

Others

$\mathbf{n}$	unit vector normal to the boundary.
∇	differential operator, 1/m

The traditional mechanical Stirling cooler uses two pistons to transfer the working fluid back and forth between hot and cold chambers that are separated by a regenerator. The heat is released from the hot chamber during the compression process and absorbed from the cold chamber during the expansion process. Efforts on a prototype micro-scale Stirling cooler, which are documented in a series of cryocooler patents [12–15], addressed frictional losses and leakage concerns by replacing the conventional pistons and the associated linkages with electrostatically-driven diaphragms.

Ceperley [16] recognized that sound waves can be used to replace the pistons or diaphragms for gas compression and displacement in the Stirling cooler. Thermoacoustic technology based on the Stirling cycle was thus developed. Reid et al. [17], Jin et al. [18], and Symko et al. [19] built thermoacoustic refrigeration

systems and applied them in cryogenic cooling. Banjare et al. [20] and Zink et al. [21] performed CFD analysis to demonstrate the principle of the thermally-driven cooling. The temperature gradient achieved due to the thermoacoustic cooling, however, is limited by the critical temperature gradient [22].

In an earlier report, we presented the design concept of a new Stirling microcooler and a first-order thermodynamic (i.e., isothermal) analysis to evaluate the system performance [23]. A parametric study showed the effects of diaphragm phase lag, swept volume ratio between the hot space and cold space, and dead volume ratio on the cooling performance. For the Stirling microcooler, modeling challenges arise mainly from the complicated geometrical structures, (e.g., the large number of pillars in the regenerator) and complex dynamics (e.g., the motion of the diaphragms). We herein address these issues.

In this paper, more detailed numerical modeling that incorporates compressible fluid flow, heat transfer, and solid mechanics is used to study the performance of the full system. COMSOL [24], a multiphysics simulation software package, is used to perform the calculations. The rest of this paper is organized as follows. In Section 2, we present our design and a review of previous work on modeling and applying the Stirling cycle, with a focus on sources of inefficiency and losses. To reduce the computational time, in Section 3 the regenerator is isolated and its design is optimized. The full system performance is then evaluated in Section 4.

2. Stirling cooler design, modeling, and inefficiencies

We recently reported on the design of a new micro-scale Stirling cooler system, which includes two diaphragms and a regenerator that separates the hot and cold chambers [23]. By the design

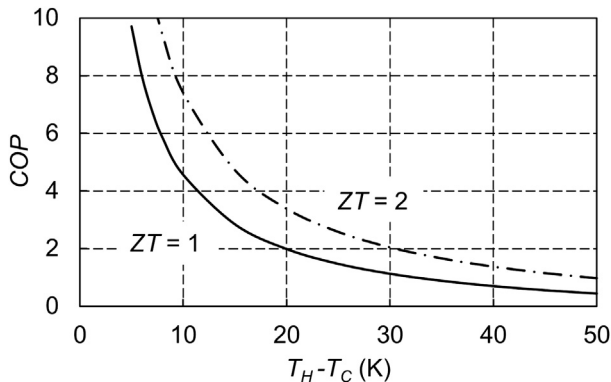


Fig. 1. Theoretical COP from Eq. (1) as a function the temperature difference ($T_H - T_C$) for a thermoelectric cooler.

requirements, each cooling element is 5 mm-long, 2.5 mm-wide, has a thickness of 0.15 mm, and is fabricated on a silicon wafer, as illustrated in Fig. 2(a) for one-half of an element. The components of one complete element are shown conceptually in Fig. 2(b). The design minimizes conduction heat losses across the 0.5 mm-long regenerator by distancing the compressor and expander assemblies with a low thermal conductivity passage, the regenerator, housing a working fluid (e.g., air) pressurized at 2 bar. The fluid flow is driven by electrostatically-actuated diaphragms. In the regenerator, an array of vertical silicon pillars serves as a thermal capacitor transferring heat to and from the working gas during the cycle. Silicon pillars are also put in the space between the chambers and the regenerator (i.e., the dead space), for improving the heat transfer between the gas and the silicon around the chambers. The pillars in the dead space are part of one continuous piece of silicon, as illustrated in Fig. 2(a). Under operating conditions, the hot and cold diaphragms oscillate sinusoidally and 90° out of phase such that the heat is extracted to the cold chamber and released from the hot chamber.

Classical theories, such as an isothermal model or an adiabatic model, can be applied to analyze Stirling coolers [25]. The isothermal analysis is based on the Schmidt cycle with sinusoidal volume variations. Assumptions are required to obtain a closed form-analytical expression for the cooling power. Notably, the gas temperature in the hot (cold) side is assumed to be the constant and to take on the temperature of the heat sink (source) and the

regenerator is assumed to be perfect. The adiabatic analysis is more realistic as the working spaces tend to be adiabatic rather than isothermal in real machines. The adiabatic analysis assumes that the hot and cold spaces are thermally-insulated and that all the heat input and output occur at the heat exchangers. The gas flows out of the heat exchangers at the heat sink (source) temperature and then comes into the hot (cold) space. The regenerator is also assumed to be perfect.

Martini [26], Wale et al. [27], Lee et al. [28], and Shoureshi [29] included more realistic losses and inefficiencies in the isothermal or adiabatic models. The important losses and inefficiencies in the Stirling cooler devices include: 1) Parasitic regenerator heat conduction, whereby a heat flow develops along the device walls due to the temperature gradient between the hot and cold sides. 2) Due to the finite convection coefficient and limited heat capacity of the regenerator, when the gas flows from the hot side to the cold side, the output gas temperature is higher than the cold space temperature, thus reducing the maximum possible heat transfer. 3) The pressure drop when the gas flows through the regenerator. 4) The finite convection coefficients in the hot and cold exchangers. For different types of Stirling devices, losses and inefficiencies may also exist due to gas leakage, mechanical damping, and electrical effects [11].

In this study, the losses associated with the regenerator (parasitic heat flow, pressure drop, and insufficient heat transfer) and the finite heat transfer in the chambers are incorporated in a system-level model to evaluate the cooling performance of the device. Future modeling work may focus on the mechanical loss associated with driving the diaphragm and electrical losses from the circuits.

3. Regenerator analysis

Estimating the COP requires knowledge of the real work input and the heat transfer inefficiencies. In our design, inefficiencies come from the convective heat transfer resistance between the silicon substrate and the gas in the chamber, the insufficient heat transfer in the regenerator (i.e., the regenerator effectiveness is not unity), and the extra work required to overcome the pressure drop through the regenerator. The regenerator is thus a critical component of the Stirling cooler. Before performing the system-level modeling, we first model the regenerator to determine how its complicated geometry can be optimized. In this part of the analysis, the gas temperatures in the chambers are assumed to be the same as the heat source/sink temperatures.

The purpose of the regenerator is to store and release heat from/to the gas during the cycling. The regenerator design has the following requirements [11]: 1) A maximum ratio of the regenerator heat capacity to the gas heat capacity. 2) A maximum heat transfer between the gas and the regenerator, requiring a large contact area. 3) A minimum pressure drop across the regenerator. 4) Complete penetration of the heat in the regenerator material when it is heated or cooled. This last requirement can be achieved by using a solid material with small characteristic dimensions. As discussed in Ref. [23], circular silicon pillars with a diameter of 20 μm are used for the regenerator solid structure due to the limitations of the heat penetration and available fabrication processes.

Finite element analysis of two-dimensional fluid flow is used to analyze the arrangement of the pillars in the regenerator (of particular interest is the porosity) in order to balance the effects of the pressure drop and heat transfer. If inefficiencies reduce the heat transfer by Q_{loss} and the work loss due to the pressure drop is W_{loss} , combining the ideal system cooling power and the work calculated by the isothermal model leads to a system COP of

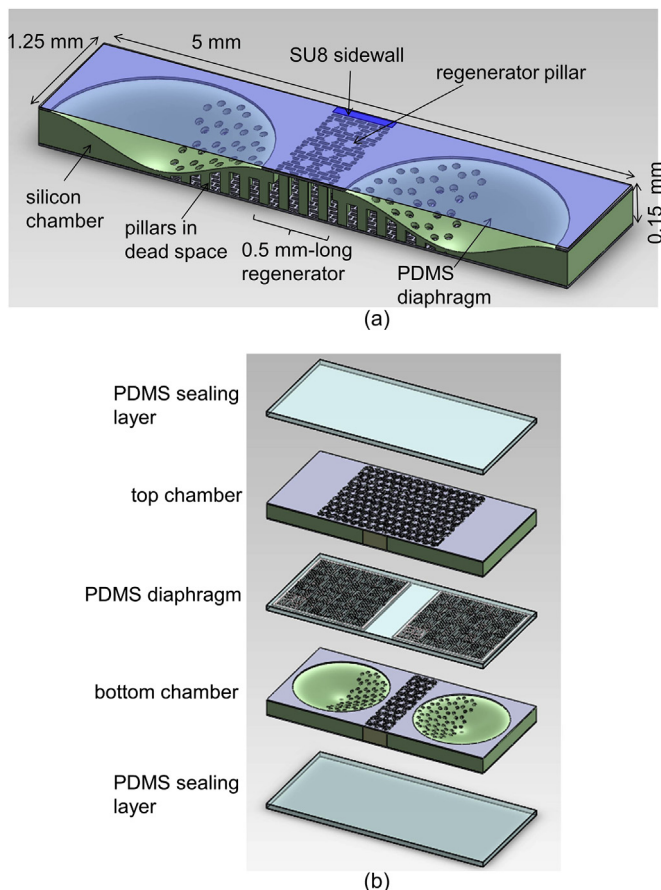


Fig. 2. Solid-model view of the Stirling microcooler element. (a) A single element is 5 mm-long, 2.5 mm-wide, has a thickness of 150 μm, and is fabricated on a silicon wafer. (b) The assembled structure has five parts: the diaphragm layer in the middle, the top and bottom chamber substrates, and two sealing PDMS layers.

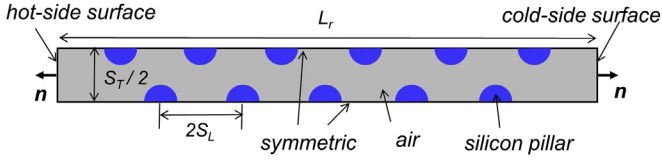


Fig. 3. Two-dimensional regenerator computational domain and boundary conditions for the porosity optimization study (see also Table 1). The horizontal pitch is S_L and the vertical pitch is S_T . The regenerator length is L_r .

$$COP = \frac{Q - Q_{\text{loss}}}{W + W_{\text{loss}}} \quad (2)$$

The computational domain and boundary conditions are illustrated in Fig. 3. Taking advantage of symmetry, we only need to consider half of an array. The horizontal and vertical pitches are S_L and S_T . The total length of the regenerator, L_r , is 0.5 mm. The width of the studied section is $S_T/2$, which corresponds to a slice of the much wider regenerator. The air flows back and forth between the hot and the cold sides. The mass flow rate at the hot-side surface is taken to be $\dot{m} = \dot{M} \sin(2\pi ft)$, where $\dot{M} = \pi \rho f S_T V_H / 2H_r$ is the mass flow rate amplitude, H_r is the regenerator height, and V_H is the volume of the hot gas space, which is 0.48 mm^3 . The volume of the chamber space is larger than the regenerator volume, which is 0.17 mm^3 when the porosity of the regenerator is unity. The no-slip condition is applied at the gas–solid interfaces. The hot-side surface temperature condition is $T = T_H = 313.15 \text{ K}$ if $\mathbf{u} \cdot \mathbf{n} < 0$ and $\nabla T \cdot \mathbf{n} = 0$ if $\mathbf{u} \cdot \mathbf{n} \geq 0$. The cold-side temperature condition is $T = T_C = 288.15 \text{ K}$ if $\mathbf{u} \cdot \mathbf{n} < 0$ and $\nabla T \cdot \mathbf{n} = 0$ if $\mathbf{u} \cdot \mathbf{n} \geq 0$. Here, $\mathbf{u}$ is the velocity vector and $\mathbf{n}$ is the unit vector normal to the boundary, as defined in Fig. 3. When the fluid flows out from the cold/hot side, the temperature is the heat source/sink temperature. Different porosities are obtained by adjusting the number of pillars, N , and their pitch. To assess the sensitivity of the results to the geometry, two S_L/S_T ratios are considered, as shown in Table 1. ϵ is the regenerator porosity. The time-averaged pressure drop across the regenerator per cycle, Δp , and the actual cold-side surface temperature, T_{out} , are obtained from the simulation results. The effectiveness of the regenerator is calculated from

$$\eta = \frac{(T_H - T_{\text{out}})}{(T_H - T_C)} \quad (3)$$

3.1. Fluid compressibility

Due to the pressure variations in the Stirling cooler (1.5–2.8 bar), fluid compressibility effects may be important. Most of the air properties relevant to this study do not change significantly within this pressure range (e.g., specific heat, thermal conductivity,

and dynamic viscosity [30]). Only the density change is significant. Before performing the regenerator optimization, we first determine if compressibility effects need to be included. For the compressible flow, the air is assumed to be ideal, and the equation of state of a classical ideal gas is used to calculate density. The gas pressure as a function of time is calculated based on the isothermal model and is in the range of 1.5–2.8 bar. For the incompressible flow, the gas pressure is set to 2 bar. The gas pressure is specified at the cold-side surface of the computation domain. The operating-frequency-dependent pressure drop through the regenerator and the regenerator effectiveness are plotted in Fig. 4(a) and (b) for both the compressible and incompressible flows. From the results, the effect of compressibility on the flow and heat transfer in the regenerator is small. We therefore use an incompressible flow computation for the optimization analysis of the regenerator.

3.2. Porosity optimization

With $T_H = 313.15 \text{ K}$ and $T_C = 288.15 \text{ K}$, the Carnot COP is 11.5. The regenerator pressure drop and effectiveness as a function of the porosity and operating frequency are plotted in Fig. 5(a) and (b). The work loss per cycle is

$$W_{\text{loss}} = 2\Delta p \cdot V_C \quad (4)$$

The insufficient heat transfer loss per cycle is

$$Q_{\text{loss}} = (1 - \eta) \times (c_p m)_g \times (T_H - T_C) \quad (5)$$

Here, m is the maximum gas mass in the hot chamber. Using Eq. (2), the actual COP is plotted in Fig. 6(a) and (b) for two different pitch ratios. The results in Fig. 5 indicate that the pressure drop increases with decreasing porosity and increasing frequency, and

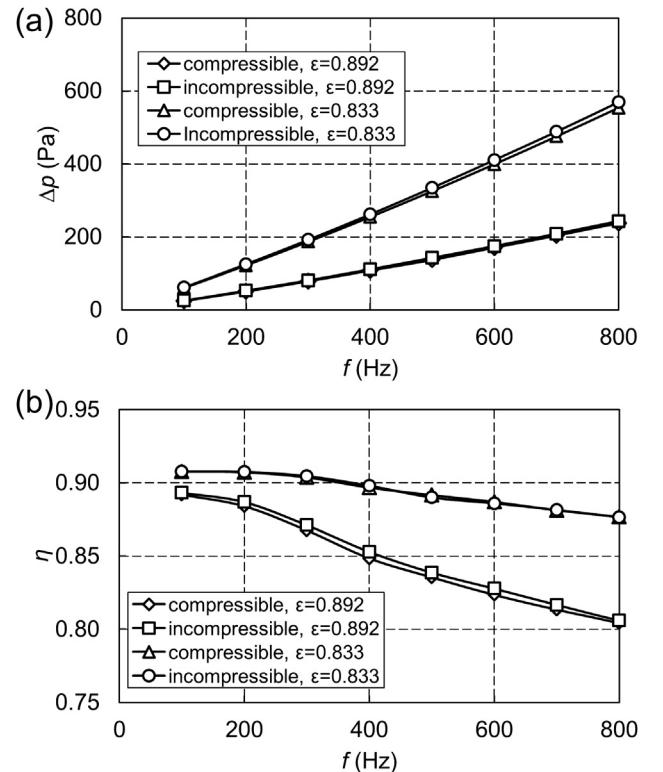


Fig. 4. Frequency-dependence of (a) time-averaged pressure drop across the regenerator and (b) regenerator effectiveness for $S_L/S_T = 0.289$ and $\epsilon = 0.892$ and 0.833 . The differences between considering compressible or incompressible flow are small.

Table 1
Regenerator pillar configurations used in porosity optimization study.

S_L/S_T	N	$S_T/2 \text{ (}\mu\text{m)}$	ϵ
0.289	13	65.7	0.938
	15	56.3	0.916
	17	49.3	0.892
	19	43.8	0.864
	21	39.4	0.833
	23	35.8	0.798
	25	32.7	0.768
0.866	8	37.5	0.933
	9	32.8	0.914
	10	29.2	0.892
	11	26.3	0.868
	12	23.9	0.842
	13	21.9	0.813
	15	19.0	0.778

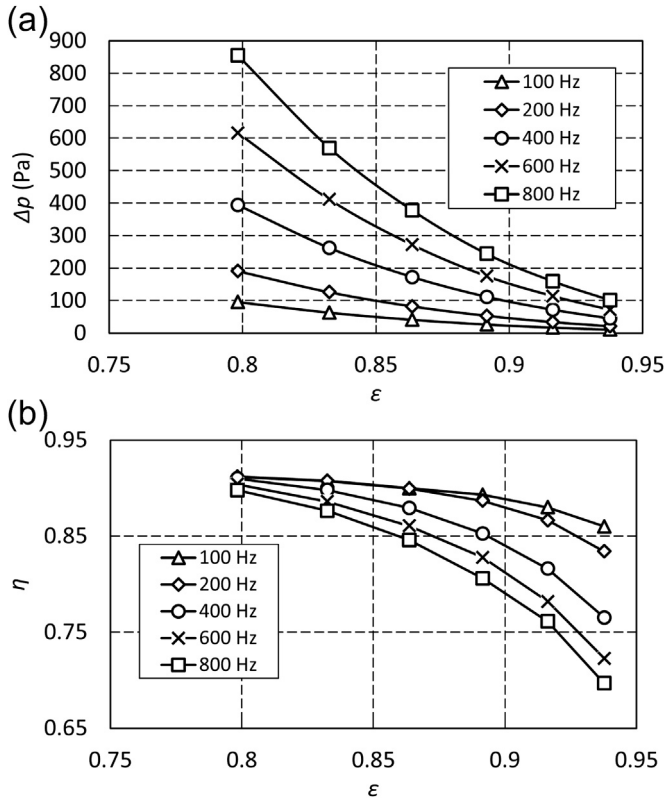


Fig. 5. Effect of operating frequency and porosity for $S_L/S_T = 0.289$ on (a) time-averaged pressure drop across the regenerator and (b) regenerator effectiveness.

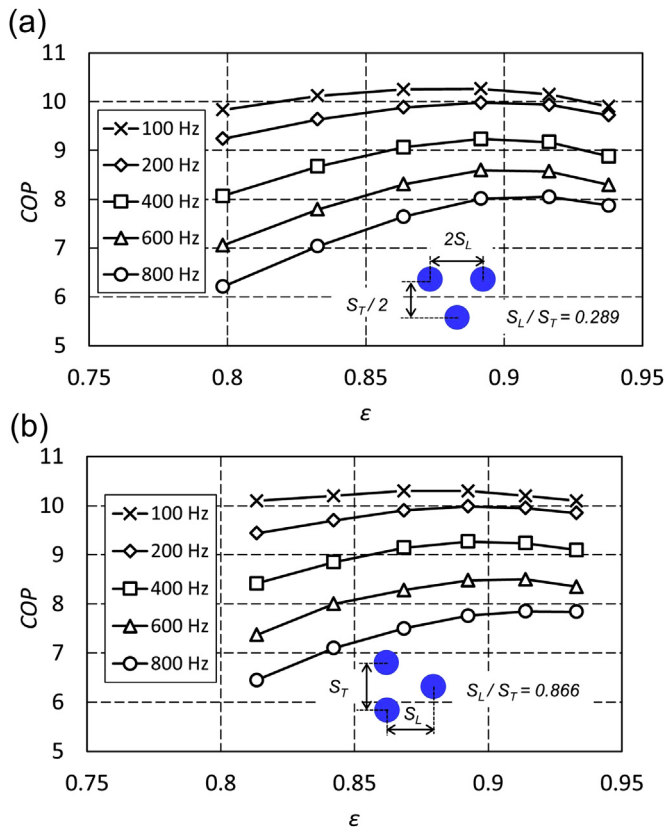


Fig. 6. Predicted COP as a function of operating frequency and porosity for (a) $S_L/S_T = 0.289$ and (b) $S_L/S_T = 0.866$.

that the heat transfer between the solid and the gas decreases with increasing porosity and frequency. An optimal porosity, as seen in Fig. 6, exists due to the large work loss at small porosities and the large heat transfer loss at high porosities. Even though the arrangements of the regenerator are significantly different ($S_L/S_T = 0.289$ and 0.866), the optimal porosity is always near 0.9. Therefore, the key parameter in the regenerator that affects the system COP is the porosity. We note that the COP at $S_L/S_T = 0.289$ is slightly bigger than that at $S_L/S_T = 0.866$ at higher frequency. As the gas temperatures in the chambers are assumed to be the same as the heat source/sink temperatures, meaning that the convection coefficients in the chambers are taken to be infinite, the COPs obtained here are overestimated.

4. System evaluation

In the previous section, the micro-Stirling cooler regenerator was modeled and we found that the optimal porosity is near 0.9. In this section, we choose a porosity of 0.892 at $S_L/S_T = 0.289$ as a case study and evaluate the full system performance. It is not computationally feasible to build a model that includes the full details of the regenerator geometry (i.e., all the pillars). To overcome the computational complexity brought about by the fine pillar structure in the regenerator and in the dead space, a porous medium model was used to replace the pillars, allowing for a full-system finite element calculation.

4.1. Porous medium governing equations

As the Stirling cooler works by gas expansion and compression processes, the main physics in the system are the compressible laminar flow and heat transfer. In the porous medium region, a non-equilibrium model is used [31]. The governing equations for the gas phase are

$$\frac{\partial \rho}{\partial t} + \frac{\nabla \cdot (\rho \mathbf{u})}{\epsilon} = 0, \quad (6)$$

$$\frac{\rho}{\epsilon} \left[\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \frac{\mathbf{u}}{\epsilon} \right] = -\nabla p - \frac{\nabla \cdot \bar{\tau}}{\epsilon} - \left(\frac{\mu}{\kappa} + \beta_F |\mathbf{u}| \right) \mathbf{u}, \quad (7)$$

$$\rho c_p \left[\epsilon \frac{\partial T}{\partial t} + (\mathbf{u} \cdot \nabla) T \right] = \nabla \cdot (k \nabla T) + \frac{\bar{\tau} : \nabla \mathbf{u}}{\epsilon} + \epsilon \frac{Dp}{Dt} + h_{fs} a_{fs} (T_s - T). \quad (8)$$

Here p is the pressure, μ is the dynamic viscosity, κ is the permeability, β_F is the Forchheimer coefficient, T_s is the solid temperature, h_{fs} is the convection coefficient between the solid and the fluid, and a_{fs} is the solid surface area per unit volume, and $\bar{\tau}$ is the viscous tensor,

$$\bar{\tau} = \mu \left[\nabla \mathbf{u} + (\nabla \mathbf{u})^T \right] - \frac{2}{3} \mu (\nabla \cdot \mathbf{u}). \quad (9)$$

Assuming the gas to be ideal, the state equation is

$$\rho = \frac{p M_g}{RT}, \quad (10)$$

where R is the ideal gas constant and M_g is the molar mass.

In the solid phase, the energy equation is

$$(1 - \epsilon) \rho_s c_{ps} \frac{\partial T_s}{\partial t} = \nabla \cdot \left(\bar{k}_e \cdot \nabla T_s \right) - h_{fs} a_{fs} (T_s - T), \quad (11)$$

where ρ_s and c_p are the density and specific heat of the solid. $\bar{k}_e$ is the effective thermal conductivity tensor of the solid phase in the porous medium.

4.2. Specifying the permeability and Forchheimer coefficient

The permeability κ and Forchheimer coefficient β_F can be obtained based on the one-dimensional Darcy–Forchheimer equation, which is [32]

$$-\frac{\partial p}{\partial x} = \frac{\mu}{\kappa} u + \beta_F u^2. \quad (12)$$

The relationship between the pressure drop and the velocity for the regenerator with pillars is obtained by steady-state simulation and is plotted in Fig. 7. The permeability and Forchheimer coefficient are then calculated by the least squares method according to Eq. (12), which is also plotted in Fig. 7. The best-fit permeability is $9.77 \times 10^{-11} \text{ m}^2$ and the best-fit Forchheimer coefficient is 9823 kg/m^4 .

4.3. Convection coefficient

The convection coefficient of the regenerator is also evaluated from steady-state simulations. The Nusselt number and Reynolds number are defined as

$$Nu_{D_h} = \frac{h_f D_h}{k}, \quad (13)$$

$$Re_{D_h} = \frac{\rho \bar{u} D_h}{\mu}, \quad (14)$$

where $D_h = 4\epsilon/a_{fs}$ is the hydraulic diameter and $\bar{u}$ is the average velocity through the regenerator. The flow in the system is laminar as the Reynolds number in the regenerator is smaller than 200. The Nusselt number is calculated using the log-mean temperature difference when a constant temperature is applied to the pillar walls and is plotted in Fig. 8. Fitting the raw data gives $Nu_{D_h} = 13.389 Re_{D_h}$ which is used for the system modeling. Some available experimental correlations of the Nusselt number for a tube bank are plotted in Fig. 8 [33–35]. These correlations focus on the flow in a larger Reynolds number range ($Re_{D_h} > 300$ in the present study). The convection coefficient is also influenced by the arrangement of pillars in the regenerator. Thus, differences between the calculation results and correlations are observed.

4.4. Computational domain and boundary conditions

The computational domain for the full system model is shown in Fig. 9. We use the porous medium to replace the regenerator

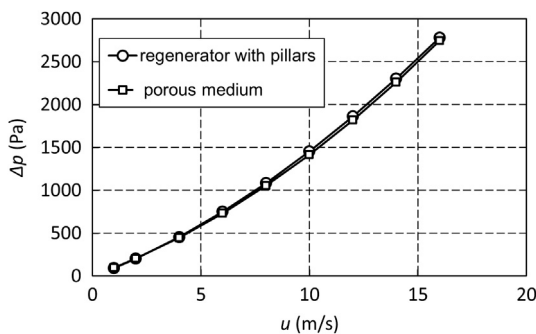


Fig. 7. Pressure drop across the regenerator as a function of average fluid velocity between (i) the full pillar structure and (ii) the porous medium model.

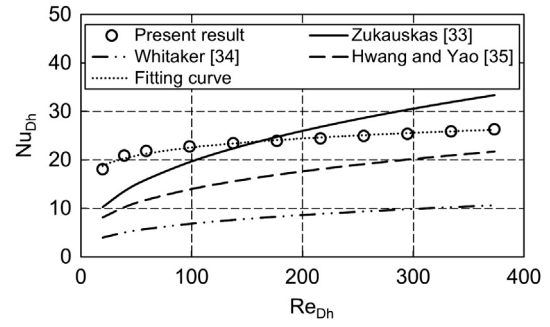


Fig. 8. Regenerator Nusselt number as a function of Reynolds number for the current calculations and from existing correlations [33–35]. The current simulation results are used for the system modeling.

and the dead-space structure. For the solid phase of the porous medium in the dead space, an isotropic thermal conductivity $(1 - \epsilon)k_{\text{silicon}}$ is assumed. The thermal conductivity of the solid phase in the regenerator is anisotropic due to the vertical array of silicon pillars, as illustrated in Fig. 2(a). The out-of-plane thermal conductivity is the same as that of the porous medium of the dead space. The in-plane thermal conductivity is taken to be that of air.

As the diaphragms are driven electrostatically, their actual motions are complicated and require detailed study. To simplify the model and the analysis, a sinusoidal motion with a 90° phase lag is applied for the cold and hot diaphragms. The displacement of the cold diaphragm is

$$z(x, y, t) = Z_{\max(x,y)} \sin(2\pi ft). \quad (15)$$

The displacement of the hot diaphragm is

$$z(x, y, t) = Z_{\max(x,y)} \sin\left(2\pi ft - \frac{\pi}{2}\right), \quad (16)$$

where f is the operating frequency and $Z_{\max(x,y)}$ is the vibration amplitude of the diaphragm. We assume that the gas space in the chamber is a spherical cap. The cap base is a circle with a 2.25 mm diameter and the maximum height of the cap is 120 μm . So,

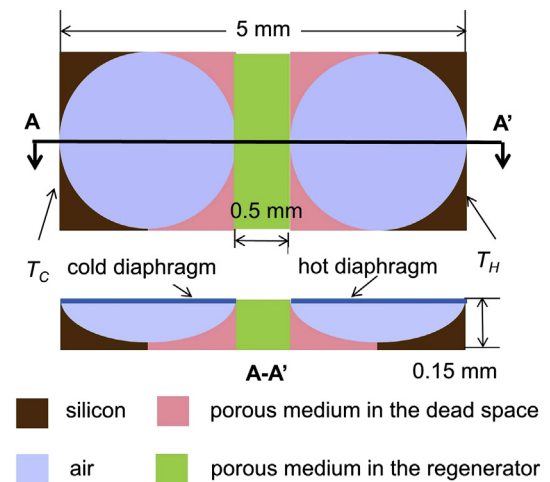


Fig. 9. Computational domain and geometry for the three-dimensional full-system model. The hot and cold diaphragms, which have a 90° phase lag, prescribe the moving boundaries of the fluid flow. The regenerator and the dead space are replaced by the porous medium.

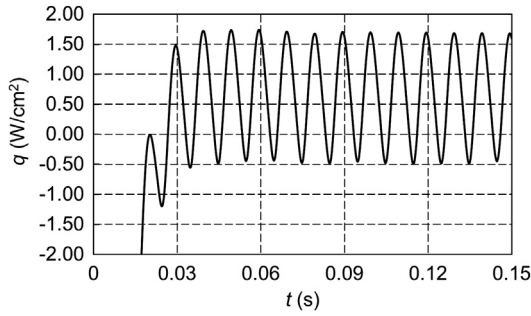


Fig. 10. Space-averaged cooling capacity as a function of time at an operating frequency of 100 Hz when the porosity is 0.892. Steady state is achieved after four cycles.

$Z_{\max}(x,y)$ represents the cap height at different positions. The hot and cold diaphragms prescribe the moving boundaries of the fluid flow. The arbitrary Lagrangian–Eulerian (ALE) moving-mesh method was used to handle the gas domain where there is a mesh deformation. The no-slip condition is applied to the solid–gas interfaces. The initial pressure in the system is 2 bar. The temperature of the end surface of the cold side is constant and is $T_C = 288.15$ K. The end surface in the hot side has a constant temperature of $T_H = 313.15$ K. The remaining surfaces are thermally insulated. The initial temperature for the whole region is 293.15 K. The number of mesh elements for the system model is 347,773. Parallel computation was performed by using 4 nodes, each of which has 16 CPUs and 16 GB of memory. Each

computation took about 20 h. The solver used for the computation is MUMPS [36].

4.5. Results and discussions

The effect of the operating frequency on the cooling capacity and COP is now explored. The frequency range is 100–800 Hz. Steady-state is reached after four cycles of the diaphragm operation at an operating frequency of 100 Hz, as shown in Fig. 10. At steady-state, the temperature contours of the gas in the system and the silicon around the chamber at one quarter and three quarters of one cycle are shown in Fig. 11(a) and (b) when the frequency is 100 Hz. In Fig. 11(a), the cold diaphragm is completely compressed and the hot gas space has its highest temperature at this point in the cycle. During the compression process, the heat is released from the hot gas and transferred to the heat sink. In Fig. 11(b), the cold gas space is expanded. At this time, the cold gas space has its lowest temperature. This process is an expansion in which the heat from the heat source is absorbed by the gas in the cold space. Through cyclings, the heat is transferred to the heat sink and absorbed from the heat source continually, as we require. The temperature contours at one quarter and three quarters of one cycle at a frequency of 800 Hz are shown in Fig. 11(c) and (d). The heat transfer process is similar compared to the results at 100 Hz. However, the maximum temperature in the hot gas space is much higher and the minimal temperature in the cold gas is much lower at 800 Hz. The average cold and hot gas space temperatures at different frequencies are plotted in Fig. 12. The cold gas temperature is colder

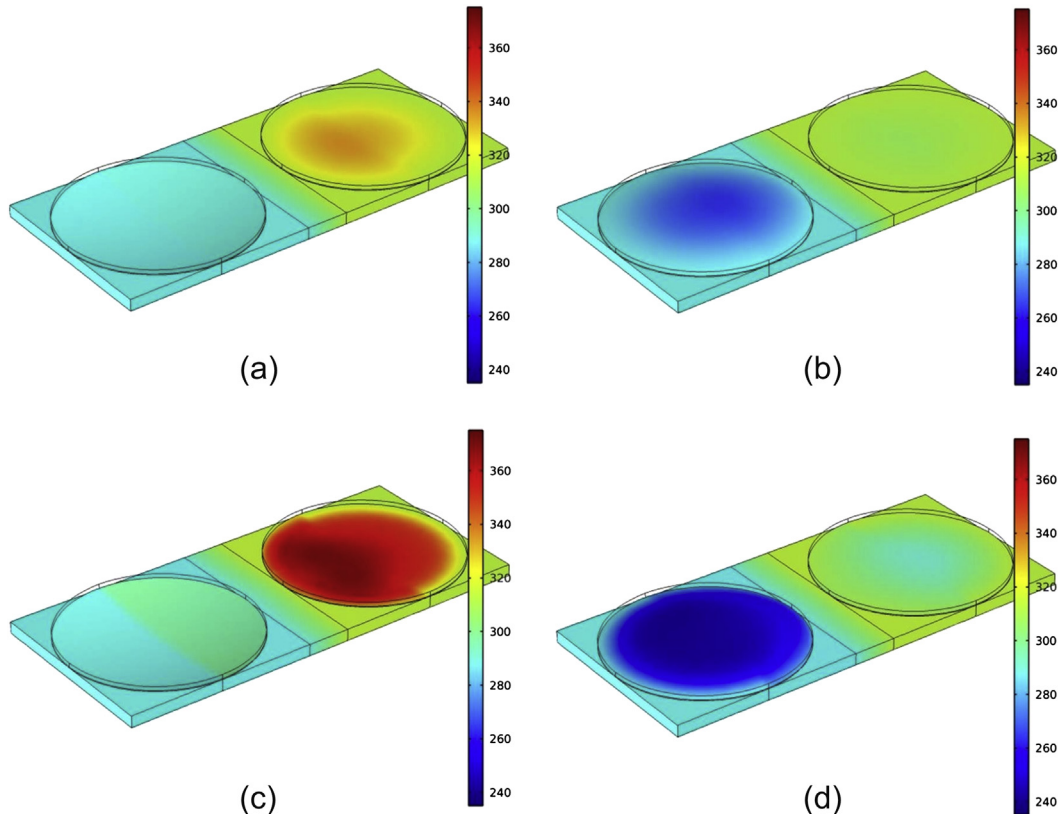


Fig. 11. Temperature contours for the full Stirling cooler system simulation at: (a) One quarter of the cycle at an operating frequency of 100 Hz. (b) Three quarters of the cycle at an operating frequency of 100 Hz. (c) One quarter of the cycle at an operating frequency of 800 Hz. (d) Three quarters of the cycle at an operating frequency of 800 Hz.

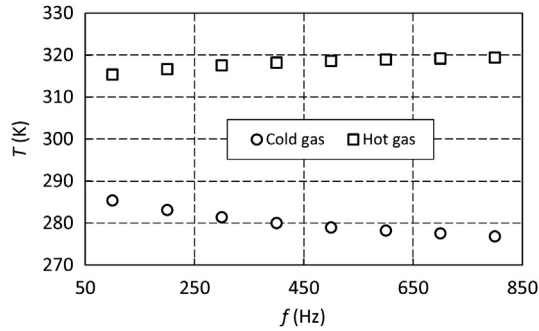


Fig. 12. Time- and space-averaged gas temperature in the cold and hot chambers after the system comes to the steady state as a function of frequency.

when the frequency is higher, and the hot gas temperature has the opposite trend. This result indicates that the heat transfer resistance between the gas and the silicon around the chamber affects the system performance, and this heat transfer resistance increases with an increase of the frequency.

The heat flux coming into the cold side from the end surface represents the cooling capacity of the Stirling cooler. The time-dependent pressure and volume variations in the element are obtained from this simulation. The PV work can therefore be calculated and the COP can be evaluated. The results of cooling capacity are shown in Fig. 13. The theoretical cooling capacities of the isothermal model and the adiabatic model are also provided in Fig. 13. The adiabatic model result is closer to the simulation as the isothermal assumption in chambers is not good due to the high operating frequency, as shown in Fig. 11. The cooling capacity increases almost linearly from 0.7 to 5.1 W/cm^2 when the frequency increases from 100 Hz to 800 Hz. The COP with respect to the cooling capacity is plotted in Fig. 14. The COP decreases from 7.4 to 2.4 when the operating frequency and the cooling capacity increase. The reason is that even though the cooling power increases when the frequency increases, the thermodynamic work in the system increases more quickly, making the COP lower. An appropriate frequency should be considered according to the requirement of the real situation. When the operating frequency is 600 Hz, the COP is 2.93, which is about 25% of the Carnot COP , and the cooling capacity is 4.2 W/cm^2 . In a standard design of a thermoelectric cooler (i.e., the material thickness is larger than 1 mm), the maximum cooling capacity is about 2 W/cm^2 and the best COP is 2.85 when the ZT of the material is 1 and the temperature difference is 15 K [2].

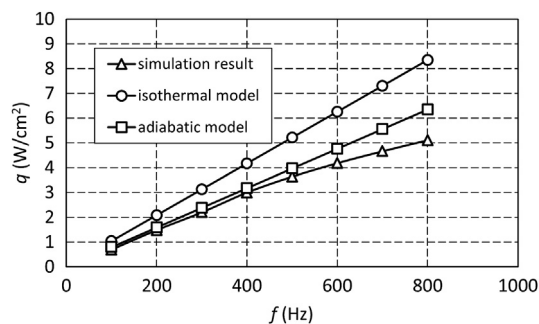


Fig. 13. System cooling capacity as a function of the frequency from the multiphysics simulations, the isothermal model, and the adiabatic model. With an increase of the operating frequency, the cooling capacity increases. The adiabatic model result is closer to the simulation result than the isothermal model.

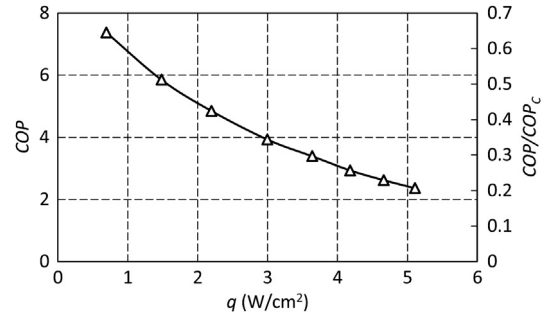


Fig. 14. Multiphysics simulation prediction of the system COP as a function of the cooling capacity. The theoretical COP is 11.5 with $T_H = 313.15$ K and $T_C = 288.15$ K.

5. Conclusions

In this paper, a new Stirling micro-scale cooler has been modeled and simulated numerically. The computations of multiphysics processes, incorporating compressible fluid flow, heat transfer, porous medium, solid mechanics, and moving mesh, have been successfully implemented.

The regenerator performance in the Stirling cooler was studied and the effects of pressure drop and heat transfer were analyzed. Optimizations indicated that the optimal porosity of the regenerator is near 0.9. The complete system modeling predicted the system-level thermal performance. Parametric studies of the design demonstrated the effect of the operating frequency on the cooling capacity and the COP of the system. When the system is operated at 600 Hz, the cooling capacity is 4.2 W/cm^2 and the system COP is 2.93 when the temperatures of the heat sink and heat source are 313.15 K and 288.15 K.

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